A-LEVEL

MATHEMATICS

7357/2

Paper 2

Mark scheme

June 2019

Version: 1.0 Final

196A7357 2/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

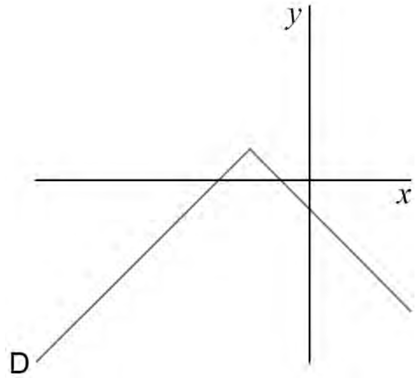
Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	2.2a	R1	
	Total		**1**	

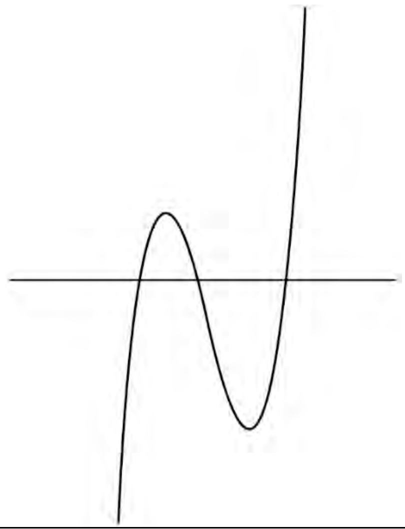
Q	Marking instructions	AO	Mark	Typical solution
2	Circles the correct response	1.1b	B1	$a^{\frac{8}{15}}$
	Total		**1**	

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.2	B1	$f(x) = x^2$
	Total		**1**	

Q	Marking instructions	AO	Mark	Typical solution
4	Explains how the factor theorem applies with reference to $f(-2) = 0$ for either function or Explains that either quadratic expression can be factorised in the form $(x + 2)(x + p)$ as $(x + 2)$ is a factor or Explains that on division by $(x + 2)$ the remainder would be zero	2.4	E1	As $(x+2)$ is a factor, then when $x=-2$, $f(x) = 0$ $4-2b+c=0$ $4-2d+e=0$ $4-2b+c=4-2d+e$ $2d-2b=e-c$ $2(d-b)=e-c$
	Uses the factor theorem with $x=-2$ substituted into one of the expressions to obtain a correct expression **NB** It is not necessary to equate to zero for this mark or Expands one of their factorised forms and equates coefficients correctly $(x+2)(x+p)=x^2+(p+2)x+2p$ $p+2=b$ $2p=c$ or Divides one of the expressions by $(x + 2)$ to obtain a correct remainder. Either one of $4-2b+c$ $4-2d+e$	1.1a	M1	
	Deduces both correct equations using factor theorem or division $4-2b+c=0$ $4-2d+e=0$ **PI** by $4-2b+c=4-2d+e$ or Expands both of their factorised forms and equates coefficients to deduce the correct equations – must not use p in both	2.2a	A1	
	Forms a single equation for b, c, d and e and completes rigorous argument to show the required result **NB** R1 can be awarded even if E1 was **not** awarded	2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Mark	Typical solution
5	Separates the variables – one side correct Condone missing integral signs **PI** by correct integration	3.1a	M1	$\int \frac{1}{x^2}\ln x\,\mathrm{d}x = \int t\,\mathrm{d}t$
	Integrates their $\int t\,\mathrm{d}t$ correctly	1.1b	A1F	$\int t\,\mathrm{d}t = \frac{t^2}{2} + c$
	Obtains $u' = \frac{1}{x}$ and $v = -\frac{1}{x}$ **OE**	1.1b	B1	$u = \ln x$ $u' = \frac{1}{x}$
	Integrates $\int \frac{1}{x^2}\ln x\,\mathrm{d}x$ Substitutes their u, u', v and v' into the correct formula for integration by parts Condone sign errors in formula	1.1a	M1	$v' = x^{-2}$ $v = -x^{-1}$ $-\frac{1}{x}\ln x - \int \frac{1}{x}\left(-x^{-1}\right)\mathrm{d}x$ $-\frac{1}{x}\ln x + \int \frac{1}{x^2}\mathrm{d}x$
	Obtains $-\frac{1}{x}\ln x - \frac{1}{x}$	1.1b	A1	$-\frac{1}{x}\ln x - \frac{1}{x}$
	Substitutes $t = 2$ and $x = 1$ into their integrated equation to find their $+c$	1.1a	M1	$-\frac{1}{x}\ln x - \frac{1}{x} = \frac{t^2}{2} + c$ $t = 2, x = 1 \Rightarrow -1 = 2 + c$
	Obtains correct solution must have $t^2 = \ldots$ **ACF**	2.5	A1	$c = -3$ $t^2 = 6 - 2\left(\frac{1 + \ln x}{x}\right)$
	Total		**7**	

Q	Marking instructions	AO	Mark	Typical solution
6	Compares with $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ by forming an identity e.g. $R\sin(x+\alpha) \equiv a\sin x + b\cos x$ **OE** or Differentiates correctly and equates to zero **CAO PI** by $a\cos x = b\sin x$ **PI** by *R* = 4 or $a^2 + b^2 = 16$	3.1a	M1	$R\sin(x+\alpha) = a\sin x + b\cos x$ $R = 4$ $4\sin\left(\frac{\pi}{3} + \alpha\right) = 2\sqrt{3}$ $\alpha = \frac{\pi}{3}$ $a = 4\cos\frac{\pi}{3} = 2$ $b = 4\sin\frac{\pi}{3} = 2\sqrt{3}$
	Deduces *R* = 4 or $a^2 + b^2 = 16$	2.2a	A1	
	Forms a correct equation for α **PI** by correct α or Forms the equation shown below $2\sqrt{3} = \frac{a\sqrt{3}}{2} + \frac{b}{2}$ **OE** **Must** substitute correct exact values for the trig functions	1.1b	B1	
	Solves their equation to obtain any correct value of α Correct values are shown below $\alpha = \frac{\pi}{3}$ or 0 for $R\sin(x \pm \alpha)$ $\alpha = \pm\frac{\pi}{6}$ for $R\cos(x \pm \alpha)$ or Eliminates a variable correctly from their two equations – **must** obtain a correct simplified equation	1.1a	M1	
	Deduces $a = 2$	2.2a	R1	
	Deduces $b = 2\sqrt{3}$	2.2a	R1	
	Total		**6**	

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Sketches any cubic graph, crossing the x-axis in three places	1.2	B1	
	Sketches any cubic graph with a positive coefficient of x^3	1.2	B1	
7(b)(i)	Differentiates to obtain $f'(x)$ Two terms with at least one correct - either $3x^2$ or $6px$	1.1a	M1	For a turning point $f'(x) = 0$ $f(x) = x^3 + 3px^2 + q$ $f'(x) = 3x^2 + 6px$
	Solves $3x^2 + 6px = 0$ to obtain $x = 0$ or $x = -2p$ or Substitutes $x = 0$ in $f'(x) = 3x^2 + 6px$ and obtains 0	1.1b	A1	$3x^2 + 6px = 0$ $3x(x + 2p) = 0$ $x = 0$ $x = -2p$
	Obtains the correct two roots $x = 0$ and $x = -2p$ **OE** and states why there must be a turning point referring to root $x = 0$	2.4	R1	Since one of the roots is $x = 0$ there must be a turning point on the y axis
7(b)(ii)	Deduces that turning point at $x = -2p$ is a maximum or deduces that turning point $x = 0$ is a minimum May have been seen in part **(b)(i)** Accept a sketch showing correct relative positions of turning points	2.2a	B1	Since $p > 0$ $x = -2p$ is the maximum $x = 0$ is the minimum $f(0) = q$ $f(-2p) = (-2p)^3 + 3p(-2p)^2 + q$ $= 4p^3 + q$
	Substitutes their $x = -2p$ into $f(x)$	1.1a	M1	
	Obtains correct $f(0) = q$ and $f(-2p) = 4p^3 + q$	1.1b	A1	$-4p^3 < q < 0$
	Deduces either $q < 0$ or $-4p^3 < q$ Condone $\leq$	2.2a	R1	
	Deduces $-4p^3 < q < 0$ **CAO**	2.2a	R1	
	Total		**10**	

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Takes logs of both sides of the equation and applies addition rule	1.1a	M1	$\log_{10} V = \log_{10} p\,q^t$ $\log_{10} V = \log_{10} p + \log_{10} q^t$ $\log_{10} V = \log_{10} p + t\log_{10} q$
	Completes rigorous argument to show required result Condone missing base	2.1	R1	
8(b)	Equates $\log_{10} p$ to 3.90 or Forms two simultaneous equations using points from the line of best fit only	3.4	M1	$\log_{10} p = 3.90$ p = 7940 $log\ q = \dfrac{5.28 - 3.90}{40 - 0} = 0.0345$ $q = 1.08$
	Calculates gradient and equates to $\log_{10} q$ or Solves their pair of simultaneous equations to obtain p and q	3.4	M1	
	Obtains correct **AWRT** 8000 **CSO**	1.1b	A1	
	Obtains correct q **AWRT** 1.1 **CSO**	1.1b	A1	
8(c)	Substitutes V= 500000 into their $V = 7940 \times 1.08^t$ or into their $\log_{10} V = \log_{10} 7940 + t\log_{10} 1.08$ to form an equation for t **PI** by correct t value	3.4	M1	$500000 = 7940 \times 1.08^t$ $t = 53.82$
	Solves their equation for t Must have $t > 40$	1.1a	M1	
	States their correct year using 1970+ their integer part of t Must be later than 2010	3.2a	A1F	The house will first be worth half a million pounds during 2023
8(d)	Explains that their 2023 (**FT** later than 2010) is outside the range of data collected	3.5b	E1F	The model is only based on data between 1970 and 2010
	Explains that house prices may not continue to grow in the same way Must refer to context not just to extrapolation/pattern Can be implied by comments such as: Theresa may have made improvements by adding a new room Prices could fall in a market crash	3.2b	E1	House prices may not continue to grow in the same way indefinitely
	Total		**11**	

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Write in a form to which the binomial expansion can be applied Must be of form $a\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$	3.1a	M1	$\sqrt{4-2x^2}=2\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$ $\approx 2\left(1+\frac{1}{2}\left(-\frac{x^2}{2}\right)\right)$ $\approx 2-\frac{x^2}{2}$
	Completes rigorous argument to obtain correct expansion **AG**	2.1	R1	
9(b)	Compares their $\frac{x^2}{2}$ to 1 Condone incorrect inequality **PI** by $\left\|-2x^2\right\|<4$	1.1a	M1	$\left\|-\frac{x^2}{2}\right\|<1$ $\Rightarrow \|x\|<\sqrt{2}$
	Obtains correct range of values **ACF**	1.1b	A1	
9(c)	Explains that as 0.4 radians is small therefore $\cos x\approx 1-\frac{x^2}{2}$ **Must** refer to 0.4 and small angle approximation for cos x	2.4	E1	As 0.4 is small $\cos x\approx 1-\frac{x^2}{2}$ $\int_0^{0.4}\sqrt{\cos x}\,dx\approx\int_0^{0.4}\sqrt{1-\frac{x^2}{2}}\,dx$ $\approx\frac{1}{2}\int_0^{0.4}2-\frac{x^2}{2}\,dx$ $\approx\int_0^{0.4}1-\frac{x^2}{4}\,dx$ $\approx\left[x-\frac{x^3}{12}\right]_0^{0.4}$ $\approx 0.4-\frac{0.4^3}{12}$ ≈ 0.39467
	Uses half of their expansion from **9(a)** as the integrand	1.1a	M1	
	Integrates their expression with at least one term correct	1.1a	M1	
	Obtains correct value must be at least five decimal places Condone $\frac{148}{375}$ **CAO**	1.1b	A1	
9(d)	States that 1.4 radians is not a small angle so the approximation is not valid **Must** refer to small angle approximation **and** 1.4 or State invalid as 1.4 is bigger than 0.664 **NB** 0.664 is the limiting value for approximation to be valid	2.4	E1	Since 1.4 is not a small angle the approximation is not suitable
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
10	Ticks correct box	2.2a	B1	The particle was decelerating for $12 \leq t \leq 20$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	1.1b	B1	1000 N
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Circles correct answer	1.1b	B1	-400
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	States appropriate *suvat* equation and clearly identifies $s = h, a = g$ and $u = 0$ **PI** by $v^2 = 0^2 + 2gh$ **OE**	1.1a	M1	$v^2 = u^2 + 2as$ $u = 0 \quad a = g \quad$ s=h $v^2 = 0^2 + 2gh$ $v = \sqrt{2gh}$
	Completes rigorous argument by substituting key values and rearranging correctly for v Must have used consistent signs for s and a **AG**	2.1	R1	
13(b)	Substitutes two values in $v = \sqrt{2gh}$ to find the third value **OE**	3.1b	M1	When $g = 9.8$ and $h = 18$ $v = \sqrt{2 \times 9.8 \times 18}$ = 18.8 18.8 < 20 Machine is faulty
	Obtains correct third value If finding v then accept **AWRT** 19 If finding g then accept **AWRT** 11 If finding h then accept **AWRT** 20	1.1b	A1	
	Makes an appropriate comparison for correct v, g or h and infers that the teacher's claim is correct. The comparison can be implied in their comment, eg the value of v is less than 20 or Makes an appropriate comparison using g = 10 and infers that the teacher's claim is incorrect. Their answer **must** be rounded to 20. The comparison can be implied in their comment, eg the value matches the given value of v	2.2b	R1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Finds a moment of a force about any point. Must have the form force x distance Can be awarded for $6R$ **PI** by fully correct equation	1.1b	B1	Take moments about *A* $mg \times 0.04 = 0.28g \times 0.03$ $m = 0.21$
	Forms a fully correct moments equation using the correct model Must have included g on both sides Moments about *B* gives (in metres) $0.28g(0.03) + 0.1mg =$ $0.06(0.28g + mg)$	3.3	M1	
	Solves equation to show $m = 0.21$ **AG**	1.1b	A1	
14(b)	Forms a moments equation for equilibrium of rod with correct number of terms – can use m, 0.21 or their value for m from part **14(a)** **Condone omission of *g* throughout part 14(b)**	3.1b	M1	Take moments about *A* $0.21g \times 0.04 = 0.048g \times 0.05 \times n$ $n = 3.5$ Maximum $n = 3$
	Forms a moments equation for equilibrium of rod with term involving n correct – can use m, 0.21 or their m value from **part 14(a)** FT their incorrect m Moments about B gives $0.06R = 0.00048ng + 0.1mg$	3.4	A1F	
	Obtains a fully correct moments equation with $m = 0.21$ substituted Moments about B gives $0.06(0.21g + 0.048ng) =$ $0.00048ng + 0.1(0.21)g$ Must have substituted correct expression for R	1.1b	A1	
	States $n = 3$ **CSO**	1.1b	A1	
14(c)	States an assumption about the rod Accept The mass/weight of the rod acts in the middle The rod is in limiting equilibrium **OE** The rod is rigid	3.5b	E1	The rod is uniform
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Finds $\overrightarrow{AB}$ or $\overrightarrow{CD}$ or $\overrightarrow{BC}$ or $\overrightarrow{DA}$ correctly Condone a direction error in the label $\overrightarrow{BC} = \begin{bmatrix}-130\\-840\end{bmatrix}$ $\overrightarrow{DA} = \begin{bmatrix}-180\\750\end{bmatrix}$ **OE** or Finds gradient of AB or CD or BC or DA correctly Gradient AB = CD = $\frac{9}{31}$ **OE** Gradient BC = $\frac{84}{13}$ **OE** Gradient DA = $-\frac{25}{6}$ **OE** Accept ratios $\frac{31}{9}, \frac{13}{84}, -\frac{6}{25}$ **OE** Ignore any incorrect labelling of ratios here	3.1a	M1	$\overrightarrow{AB} = \begin{bmatrix}-620\\-180\end{bmatrix}$ $\overrightarrow{CD} = \begin{bmatrix}930\\270\end{bmatrix}$ $\overrightarrow{CD} = -1.5 \times \overrightarrow{AB}$ Thus AB and CD are parallel but not equal in length $ABCD$ is a trapezium but not a parallelogram
	Finds $\overrightarrow{AB}$ and $\overrightarrow{CD}$ correctly **OE** or Finds gradients of AB and CD correctly or Finds a corresponding pair of ratios correctly – **Do not** award if reciprocals of gradients are labelled as gradients or vectors	1.1b	A1	
	Shows/states $\overrightarrow{CD} = \pm 1.5 \times \overrightarrow{AB}$ **OE** or Shows/states that $\overrightarrow{BC} \neq k \times \overrightarrow{DA}$ or Finds $\overrightarrow{BC}$ and $\overrightarrow{DA}$ correctly or Finds gradients of BC and DA correctly or Finds a second corresponding pair of ratios correctly– **Do not** award if reciprocals of gradients are labelled as gradients or vectors If incorrect labelling used for ratios then maximum mark is **M1 A0 A0 E1 R0**	1.1b	A1	

	Deduces that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel - implied by reference to equal gradients or Deduces correctly that $\overrightarrow{BC}$ and $\overrightarrow{DA}$ are not parallel **NB** E1 is Independent of any other marks	3.2a	E1	
	Completes rigorous proof by deducing correctly that the scalar multiple of ± 1.5 **OE** means the parallel sides are not equal in length or Completes rigorous proof by deducing correctly that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel giving justification and that $\overrightarrow{BC}$ and $\overrightarrow{DA}$ are not parallel giving justification Must include a statement that $ABCD$ is not a parallelogram at some point **NB** R1 can be awarded even if **E1** was **not** awarded **CSO**	2.1	R1	
15(b)	Uses velocity/displacement/time relationship Evidenced by dividing any vector /distance from part **15(a)** by 50	3.1b	M1	$\boldsymbol{v} = \frac{1}{50} \times \begin{bmatrix} -130 \\ -840 \end{bmatrix}$ $\boldsymbol{v} = \begin{bmatrix} -2.6 \\ -16.8 \end{bmatrix}$ $Speed = \lvert\boldsymbol{v}\rvert = \sqrt{2.6^2 + 16.8^2}$ $Speed = 17$ m s^{-1}
	Finds the magnitude of their $\overrightarrow{BC}$ or v	1.1a	M1	
	Obtains 17	1.1a	A1	
	States correct speed with correct units	3.2a	A1	
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	Differentiates to obtain $\frac{dv}{dt}$ with at least one exponent term correct	3.4	M1	$\frac{dv}{dt} = 10.512e^{-0.9t} - 0.009e^{0.3t}$
	Obtains fully correct expression for $\frac{dv}{dt}$	1.1b	A1	Maximum v occurs when $\frac{dv}{dt} = 0$
	Explains that maximum v occurs when $\frac{dv}{dt} = 0$ Accept reference to stationary point	2.4	E1	$10.512e^{-0.9t} - 0.009e^{0.3t} = 0$
	Forms equation $\frac{dv}{dt} = 0$ and solves to find a value for t **PI** by correct t	1.1a	M1	$t = 5.886$
	Obtains correct value of t **AWRT** 5.9	1.1b	A1	
	Substitutes their t into the given model **PI** by correct v	1.1b	M1	$v = 11.71 - 11.68e^{-0.9\times5.886}$ $-0.03e^{0.3\times5.886}$
	Finds value for maximum v **AWRT 11.5**	1.1b	A1	$v = 11.5$
	Justifies final answer as being a maximum value eg: • This is the maximum value as it is the **only** value which relates to $\frac{dv}{dt} = 0$ • Evaluates second derivative at t = 5.9 where $\frac{d^2v}{dt^2} = -9.4608e^{-0.9t} - 0.0027e^{0.3t}$ obtaining correct value of -0.063 or explains both terms are negative so it is less than 0 • Tests first derivative considering gradient either side of t=5.9 • Sketches curve with maximum identified at (5.9 , 11.5) **CSO** **NB** R1 can be awarded even if E1 was **not** awarded	2.4	R1	This is the maximum value as it is the **only** value which relates to $\frac{dv}{dt} = 0$

16(b)	Integrates at least one term correct	3.4	M1	$s = \int v\, dt$
	Integrates at least two terms correct	1.1a	M1	
	Obtains a fully correct integrated expression including a constant	1.1b	A1	$s = 11.71t + 12.978e^{-0.9t} - 0.1e^{0.3t} + c$
	Interprets initial conditions - states $s = 0$ when $t = 0$ **PI** by substitution of correct values	3.4	B1	$s = 0$ when $t = 0$
	Substitutes $s = 0$ and $t = 0$ to find their constant – must be clear evidence of substitution seen if incorrect c obtained	1.1a	M1	$c = -12.878$
	Obtains fully correct expression for distance – coefficients can be in any form and do not have to be evaluated as a single decimal **ACF**	3.2a	A1	Distance = $11.71t + 12.978e^{-0.9t} - 0.1e^{0.3t} - 12.878$
16(c)	Substitutes $t = 9.8$ into their expression for distance to find *s* **PI** by sight of 99.99 m for s or Substitutes *s* = 100 into their expression for distance to find *t* **PI** by sight of 9.801 for t	1.1a	M1	$s = 99.99$ m Model predicts distance to be 99.99 which is very near to 100 Accurate
	Compares s value with 100 metres or t value with 9.8 and concludes that it is a good model **CAO**	3.5a	A1	
	Total		**16**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)	Resolves vertically to form a three term equation Condone sign error or sin/cos error	3.1b	M1	$R + T\sin\theta = Mg$
	Obtains fully correct equation for resolving vertically	1.1b	A1	
	Uses Newton's second law horizontally to form a three term equation Condone sign error or consistent cos/sin error	3.1b	M1	$T\cos\theta - F = Ma$
	Obtains fully correct equation for resolving horizontally	1.1b	A1	$F = \mu R$ $T\cos\theta - \mu R = Ma$
	Uses $F = \mu R$ to replace F with μR in their horizontal equation	3.3	B1	$T\cos\theta - \mu(Mg - T\sin\theta) = Ma$
	Eliminates R to form a single equation	1.1a	M1	$T(\cos\theta + \mu\sin\theta) = Ma + \mu Mg$
	Completes rigorous argument to find required expression. Must see T as a factor before division e.g. $T(\cos\theta + \mu\sin\theta)$ **AG**	2.1	R1	$T = \dfrac{M(a + \mu g)}{\cos\theta + \mu\sin\theta}$
17(b)	Explains that the relationship may not be valid because the sledge is at rest	2.4	B1	The sledge is at rest so the relationship may not be valid as friction may not be acting at its limiting value
	Identifies that friction may not be at its limiting value Accept reference to $F \le \mu R$ Sledge may not be on the point of slipping	3.5b	B1	
	Total		**9**	